Answers on Line !!

Name: ANSWER KEY Date: ____________

Pre-Calculus Extra Practice - Sections 2.1-2.5

1. Given: $y = 5x^2 + 3x - 1$

a) Identify vertex and axis of symmetry for the quadratic function.

$\frac{-b}{2a}$ $\boxed{\left(\frac{-3}{10}, \frac{-29}{20}\right)}$ $y = 5\frac{9}{100} + 3\left(\frac{-3}{10}\right) - 1$

$= \frac{9}{20} - \frac{18}{20} - \frac{20}{20} = -\frac{29}{20}$

b) Rewrite the function in vertex form.

$$\boxed{y = 5\left(x + \frac{3}{10}\right)^2 - \frac{29}{20}}$$

c) Identify the transformations that were applied to $y = x^2$.

$$\boxed{L\ \tfrac{3}{10},\ V\ ST\ 5,\ D\ \tfrac{29}{20}}$$

2. Write an equation in standard form for the parabola that has a vertex of (-1,1) and passes through the point (3,-2).

$-2 = a(3+1)^2 + 1$

$-2 = 16a + 1$

$\frac{-3}{16} = a$

$\boxed{y = -\frac{3}{16}(x+1)^2 + 1}$ vertex form

$-\frac{3}{16}(x^2+2x+1) + \frac{16}{16}$

$\boxed{y = -\frac{3}{16}x^2 - \frac{3}{8}x + \frac{13}{16}}$

3. Describe the end behavior of the polynomial function $f(x) = 3x^3 - 2x^2 + 5x - 9$ using proper limit notation.

odd degree, + Leading Coef.

$$\boxed{\begin{array}{l} \lim_{x\to -\infty} f(x) = -\infty \\ \lim_{x\to \infty} f(x) = \infty \end{array}}$$

4. Identify the degree and sign of the lead coefficient of a function if the following conditions are true for the function:

- The number of local minimums is 6
- The number of local maximums is 7

$6 + 7 = 13$

$\boxed{\text{Degree } 14,\ \text{Lead Coeff } \ominus}$

5. Use long division and synthetic division to divide *f(x) by h(x).*

$f(x) = 3x^4 - 5x^3 - 2x^2 + 3x - 6$ and $h(x) = 3x + 1$

$\boxed{x^3 - 2x^2 + 1 + \frac{-7}{3x+1}}$

$$\begin{array}{r} 3x+1 \overline{)\,3x^4 - 5x^3 - 2x^2 + 3x - 6} \\ -3x^4 + {}^-x^3 \qquad\qquad\qquad \\ \hline -6x^3 - 2x^2 \qquad\quad \\ +6x^3 + 2x^2 \qquad\quad \\ \hline 3x - 6 \\ -3x + {}^-1 \\ \hline -7 \end{array}$$

$$\begin{array}{r|rrrrr} \frac{-1}{3} & 3 & -5 & -2 & 3 & -6 \\ & \downarrow & -1 & 2 & 0 & -1 \\ \hline & 3 & -6 & 0 & 3 & -7 \end{array}$$

$\div 3$

$\boxed{1x^3 - 2x^2 + 0x + 1 \quad \frac{-7}{3x+1}}$

6. Use the Factor Theorem to decide if the following are factors of $f(x)$.

a) $x-2$ of $f(x)=x^3-4x^2+8x-8$.

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 8 & -8 \\ & \downarrow & 2 & -4 & 8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

OR $f(2) = 2^3 - 4(2)^2 + 8(2) - 8$

$= 8 - 16 + 16 - 8$

$= \boxed{0}$ ✓

$\boxed{\text{YES, a factor}}$

b) $x+3$ of $f(x)=x^3+2x^2-4x-2$.

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -4 & -2 \\ & \downarrow & -3 & 3 & 3 \\ \hline & 1 & -1 & -1 & 1 \end{array}$$

$f(-3) = (-3)^3 + 2(-3)^2 - 4(-3) - 2$

$= -27 + 18 + 12 - 2$

$= \boxed{1}$ X

$\boxed{\text{No, not a factor}}$

7. Using the Remainder Theorem to identify the remainder when $f(x)=3x^4+3x^3-x+7$ is divided by $x-3$.

$f(3) = 3(3)^4 + 3(3)^3 - 3 + 7$

$243 + 81 - 3 + 7$

$\boxed{328}$

$$\begin{array}{r|rrrrr} 3 & 3 & 3 & 0 & -1 & 7 \\ & \downarrow & 9 & 36 & 108 & 321 \\ \hline & 3 & 12 & 36 & 107 & 328 \end{array}$$

8. Rewrite $f(x)=-3x^4+219x^3-627x^2-219x+630$ as a product of linear factors. (Start by using the graphing calculator and then confirm the factors with synthetic division.)

calc: $-1, 1, 3, ?$

$$\begin{array}{r|rrrrr} -1 & -3 & 219 & -627 & -219 & 630 \\ & \downarrow & 3 & -222 & 849 & -630 \\ \hline 1 & -3 & 222 & -849 & 630 & 0 \\ & \downarrow & -3 & 219 & -630 & \\ \hline 3 & -3 & 219 & -630 & 0 & \\ & \downarrow & -9 & 630 & & \\ \hline & -3 & 210 & 0 & & \end{array}$$

4th D

3rd D

2nd D

1st D

$-3x + 210 = 0$

$-3x = -210$

$\boxed{x = 70}$

$\boxed{\text{Zeros: } -1, 1, 3, 70}$

confirm

$$\begin{array}{r|rr} 70 & -3 & 210 \\ & \downarrow & -210 \\ \hline & -3 & 0 \end{array}$$ ✓

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$\boxed{\text{Linear factors: } y = 3(x+1)(x-1)(x-3)(x-70)}$

9. Given: $g(x) = x^4 - 2x^3 - x^2 - 4x + 12$

12 TOTAL

a) List all the possible rational zeroes of $g(x)$.

$$\pm\frac{p}{q} = \boxed{\frac{\pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1}{1}}$$

b) Find all the zeros of $g(x)$.

Calc: $2^{\times 2}$

4th D

$$\begin{array}{r|rrrrr} 2 & 1 & -2 & -1 & -4 & 12 \\ & \downarrow & 2 & 0 & -2 & -12 \\ \hline 2 & 1 & 0 & -1 & -6 & 0 \\ & \downarrow & 2 & 4 & 6 & \\ \hline & 1 & 2 & 3 & 0 & \end{array}$$

✓ 3rd D

✓ 2nd

$$\boxed{\text{Zeros: } 2^{\times 2}, -1 \pm i\sqrt{2}}$$

$x^2 + 2x + 3 = 0$

$$x = \frac{-2 \pm \sqrt{4 - 4(3)}}{2} = \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm 2i\sqrt{2}}{2} = \boxed{-1 \pm i\sqrt{2}}$$

c) ~~Rewrite~~ $g(x)$ as a product of linear and irreducible quadratics with real coefficients.

$$\boxed{g(x) = (x-2)^2(x^2+2x+3)}$$

d) Rewrite $g(x)$ as a product of linear factors.

$$\boxed{g(x) = (x-2)^2(x+1+i\sqrt{2})(x+1-i\sqrt{2})}$$

mult out!

10. a) Find a polynomial of degree 4 with zeroes -3(x2) and 2 + *i* and a lead coefficient of -4.

$$\boxed{-3, -3, 2+i, 2-i}$$

$$y = -4(x+3)^2(x-(2+i))(x-(2-i))$$

$$(x-2-i)(x-2+i)$$

$$(x-2)^2 - i^2$$

$$x^2 - 4x + 4 - -1$$

must mult out!!

$$y = -4(x+3)^2(x^2-4x+5)$$

$$= -4(x^2+6x+9)(x^2-4x+5) = \boxed{-4x^4 - 8x^3 + 40x^2 + 24x - 180}$$

b) What should the graph look like at the zero -3?

Since an even mult, should be tangent. Since even degree (4^{th}) with neg. leading coeff, ↙ ↘

$\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = -\infty$ so below x-axis.

11. Given: $2 + 3i$ and $-4 - 7i$

Add, subtract, multiply and divide the two complex numbers. Write all answers in standard form.

a) $2+3i + -4-7i = \boxed{-2-4i}$

b) $-(\quad) = \boxed{6+10i}$

c) $(\quad)(\quad) = -8-14i-12i-21i^2 = \boxed{13-26i}$

d $\frac{2+3i}{-4-7i}\left(\frac{-4+7i}{-4+7i}\right) = \frac{-8+2i+21i^2}{16-49i^2} = \frac{-29+2i}{65} = \boxed{\frac{-29}{65}+\frac{2i}{65}}$

multiply out!

12. Find a polynomial function with real coefficients that meets the following requirements:

a) Degree 3 zeros: $2-i$, -1 $f(2) = 6$

$2+i$

$y = a(x-(2+i))(x-(2-i))(x+1)$

$(x-2-i)(x-2+i)$

$(x^2-4x+4-i^2)(x+1)$

$y = a(x^2-4x+5)(x+1)$

$6 = a(4-8+5)(3)$

$a = 2$

$y = 2(x^2-4x+5)(x+1)$

$\boxed{y = 2x^3-6x^2+2x+10}$

b) Degree 4 zeros: $2i$, $3-i$ $f(0) = 20$

$-2i, 3+i$

$(x-2i)(x+2i)(x-3+i)(x-3-i)$

$x^2-4i^2 \qquad x^2-6x+9-i^2$

$y = a(x^2+4)(x^2-6x+10)$

$20 = a(4)(10)$

$a = \frac{1}{2}$

$y = \frac{1}{2}(x^2+4)(x^2-6x+10)$

$\boxed{y = \frac{1}{2}x^4-3x^3+7x^2-12x+20}$

13. $2 - 6i$ is a zero of $f(x) = x^4 - 4x^3 + 41x^2 - 4x + 40$. Find the remaining zeros of $f(x)$.

$2+6i$

$(x-2+6i)(x-2-6i)$

$x^2-4x+4-36i^2$

$x^2 + 1$

$x^2-4x+40 \,\big)\, \cancel{x^4-4x^3+41x^2-4x+40}$

$\cancel{x^4+4x^3+40x^2}$

$x^2-4x+40$

$2-6i \,|\, 1 \quad -4 \quad 41 \quad -4 \quad 40$

$\quad 2-6i \quad -40 \quad 2-6i \quad -40$

$2+6i \,|\, 1 \quad -2-6i \quad 1 \quad -2-6i \quad 0$

$\quad 2+6i \quad 0 \quad 2+6i$

$1 \quad 0 \quad 1 \quad 0$

$x^2+1=0$

$x^2=-1$

$\boxed{x=\pm i}$

$x^2+1=0$

$x^2=-1$

$x=\pm i$

$\boxed{\text{Zeros: } 2\pm 6i, \pm i}$