

Name Key

Pre-Calculus: Practice Problems for Sections 2.6, 2.7 and 7.3

Use interval notation to state the domain of each rational function.

1) $g(x) = \frac{8x^2}{(x+9)(x-2)}$ $x \neq -9, 2$

$(-\infty, -9) \cup (-9, 2) \cup (2, \infty)$

2) $g(x) = \frac{x+9}{x^2+16}$

$(-\infty, \infty)$

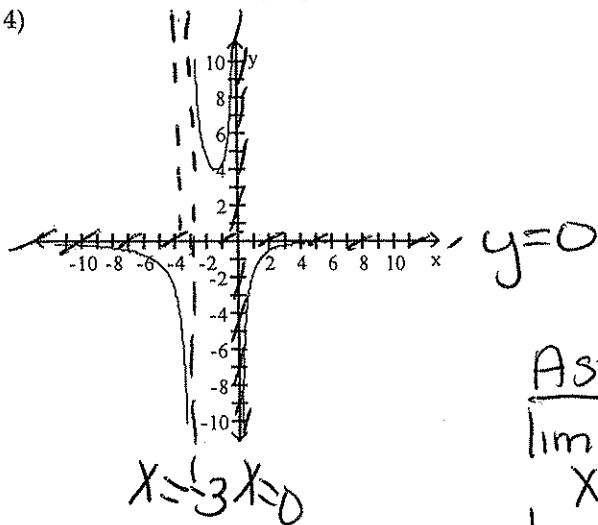
3) $h(x) = \frac{x+7}{x^2+36x}$

$x(x+36)$ $x \neq 0, -36$

$(-\infty, -36) \cup (-36, 0) \cup (0, \infty)$

State the end behavior and asymptotic behavior of each function.

4)



End Behavior

$\lim_{x \rightarrow -\infty} f(x) = 0$

$\lim_{x \rightarrow \infty} f(x) = 0$

Asymptotic Behavior

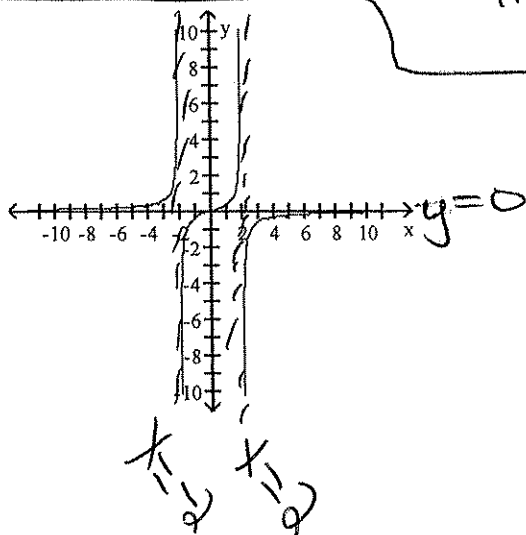
$\lim_{x \rightarrow -3^-} f(x) = -\infty$

$\lim_{x \rightarrow -3^+} f(x) = \infty$

$\lim_{x \rightarrow 0^-} f(x) = \infty$

$\lim_{x \rightarrow 0^+} f(x) = -\infty$

5)



End Behavior

$\lim_{x \rightarrow -\infty} f(x) = 0$

$\lim_{x \rightarrow \infty} f(x) = 0$

Asymptotic Behavior

$\lim_{x \rightarrow -2^-} f(x) = \infty$

$\lim_{x \rightarrow -2^+} f(x) = -\infty$

$\lim_{x \rightarrow 2^-} f(x) = \infty$

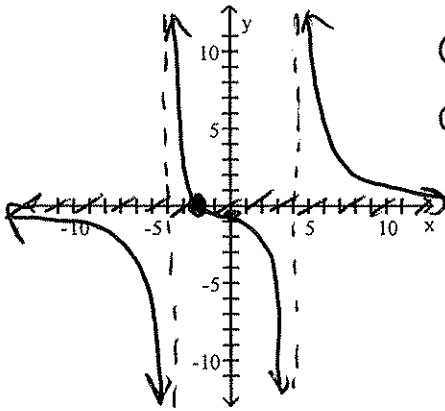
$\lim_{x \rightarrow 2^+} f(x) = -\infty$

Directions: For #6-8, identify the a) x-intercept(s), b) y-intercept, c) vertical asymptote(s), and end behavior asymptote. Graph each function.

$$6) g(x) = \frac{x+2}{x^2-16}$$

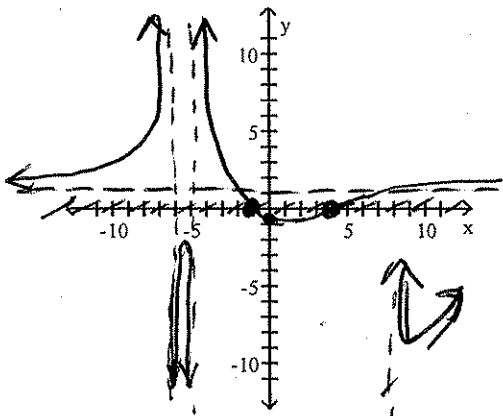
- a) X Int. $(-2, 0)$
 b) y Int. $(0, -1/8)$
 c) VA: $x=4, x=-4$
 d) EBA: $y=0$
 $n < m$

6) _____ (



$$7) g(x) = \frac{x^2 - 3x - 4}{x^2 + 11x + 30} = \frac{(x+1)(x-4)}{(x+5)(x+6)}$$

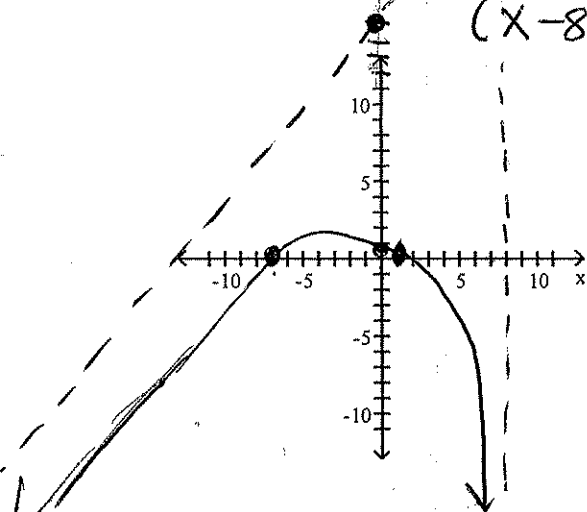
7) _____



- a) X Int. $(-1, 0)$ $(4, 0)$
 b) y Int. $(0, -2/15)$
 c) VA: $x = -5, x = -6$
 d) EBA: $y = 1$
 $n = m$

$$8) g(x) = \frac{x^2 + 6x - 7}{x - 8} = \frac{(x-1)(x+7)}{(x-8)}$$

8) _____



- a) X Int. $(1, 0)$ $(-7, 0)$
 b) y Int. $(0, 7/8)$
 c) VA: $x = 8$
 d) EBA: $y = x + 14$
 $n > m$

$$\begin{array}{r} 8 \overline{) 1 \ 6 \ -7} \\ \underline{8 \ 112} \\ 1 \ 14 \ 105 \end{array}$$

Find the end behavior asymptote of the rational function. State the end behavior.

$$9) f(x) = \frac{x^2 - 3x + 2}{x + 5}$$

$$EBA: y = x - 8$$

9) _____

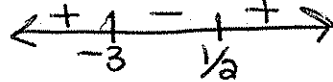
$$\lim_{x \rightarrow -\infty} f(x) = -\infty \quad \lim_{x \rightarrow \infty} f(x) = \infty$$

$$\begin{array}{r} -5 \overline{) 1 \quad -3 \quad 2} \\ \underline{ } \\ 1 \quad -8 \quad 40 \end{array}$$

Solve the polynomial inequalities algebraically. Express each solution set in interval notation.

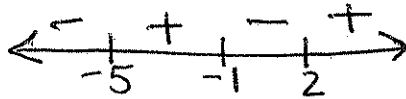
$$10) 2x^2 + 5x - 3 \leq 0$$

$$(2x - 1)(x + 3) \leq 0$$



$$10) [-3, 1/2)$$

$$11) (x + 5)(x + 1)(x - 2) > 0$$



$$11) (-5, -1) \cup (2, \infty)$$

$$12) 5x^2 - 3x \geq 8$$

$$5x^2 - 3x - 8 \geq 0$$

$$(5x - 8)(x + 1) \geq 0$$



$$12) (-\infty, -1] \cup [8/5, \infty)$$

Solve each rational inequality algebraically. Express each solution set in interval notation.

$$13) \frac{-x + 7}{x - 3} \geq 0$$



$$13) (3, 7]$$

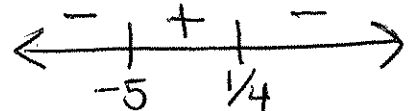
$$14) \frac{x + 26}{x + 5} < 0$$

$$\frac{x + 26 - 5}{x + 5} < 0$$

$$\frac{x + 26 - 5(x + 5)}{x + 5} < 0$$

$$\frac{x + 26 - 5x - 25}{x + 5} < 0$$

$$\frac{-4x + 1}{x + 5} < 0$$



$$14) (-\infty, -5) \cup (1/4, \infty)$$

Write the partial fraction decomposition of the rational expression.

$$15) \frac{12x - 58}{x^2 - 10x + 24} = \frac{A}{x - 6} + \frac{B}{x - 4}$$

$$15) \frac{7}{x - 6} + \frac{5}{x - 4}$$

$$\begin{aligned} 12x - 58 &= A(x - 4) + B(x - 6) \\ &= Ax - 4A + Bx - 6B \\ &= (A + B)x + (-4A - 6B) \end{aligned}$$

$$\begin{aligned} 12 &= A + B \\ -58 &= -4A - 6B \\ \rightarrow 48 &= 4A + 4B \\ \hline -10 &= -2B \\ B &= 5 \end{aligned}$$

$$\begin{aligned} 12 &= A + 5 \\ A &= 7 \end{aligned}$$

$$16) \frac{x+2}{x^3-2x^2+x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \quad x(x-1)^2$$

$$x(x^2-2x+1)$$

$$x(x-1)(x-1)$$

$$x(x-1)^2$$

$$16) \frac{2}{x} + \frac{-2}{x-1} + \frac{3}{(x-1)^2}$$

$$x+2 = A(x-1)^2 + Bx(x-1) + Cx$$

$$= A(x^2-2x+1) + Bx^2 - Bx + Cx$$

$$= Ax^2 - 2Ax + A + Bx^2 - Bx + Cx$$

$$= (A+B)x^2 + (-2A-B+C)x + (A)$$

$$0 = A+B \quad 1 = -2A-B+C \quad (2=A)$$

$$(B=-2) \quad 1 = -4+2+C$$

$$(C=3)$$

$$17) \frac{12x-7}{(x-1)^2(x^2+4)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+4}$$

$$17) \frac{2}{x-1} + \frac{1}{(x-1)^2} + \frac{-2x-3}{x^2+4}$$

Setup Only

$$12x-7 = A(x-1)(x^2+4) + B(x^2+4) + (Cx+D)(x-1)^2$$

$$= A(x^3+4x-x^2-4) + Bx^2+4B + (Cx+D)(x^2-2x+1)$$

$$= Ax^3 + 4Ax + Ax^2 - 4A + Bx^2 + 4B + Cx^3 - 2Cx^2 + Cx + Dx^2 - 2Dx + D$$

$$= (A+C)x^3 + (-A+B-2C+D)x^2 + (4A+C-2D)x + (-4A+4B+D)$$

$$0 = A+C \quad 0 = -A+B-2C+D \quad 12 = 4A+C-2D \quad -7 = -4A+4B+D$$

$$18) \frac{4x^3-5x^2+3x-5}{(x^2+2)^3} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2} + \frac{Ex+F}{(x^2+2)^3}$$

$$18) \frac{4x-5}{(x^2+2)^2} + \frac{-5x+5}{(x^2+2)^3}$$