Complete Graphs of Rational Functions

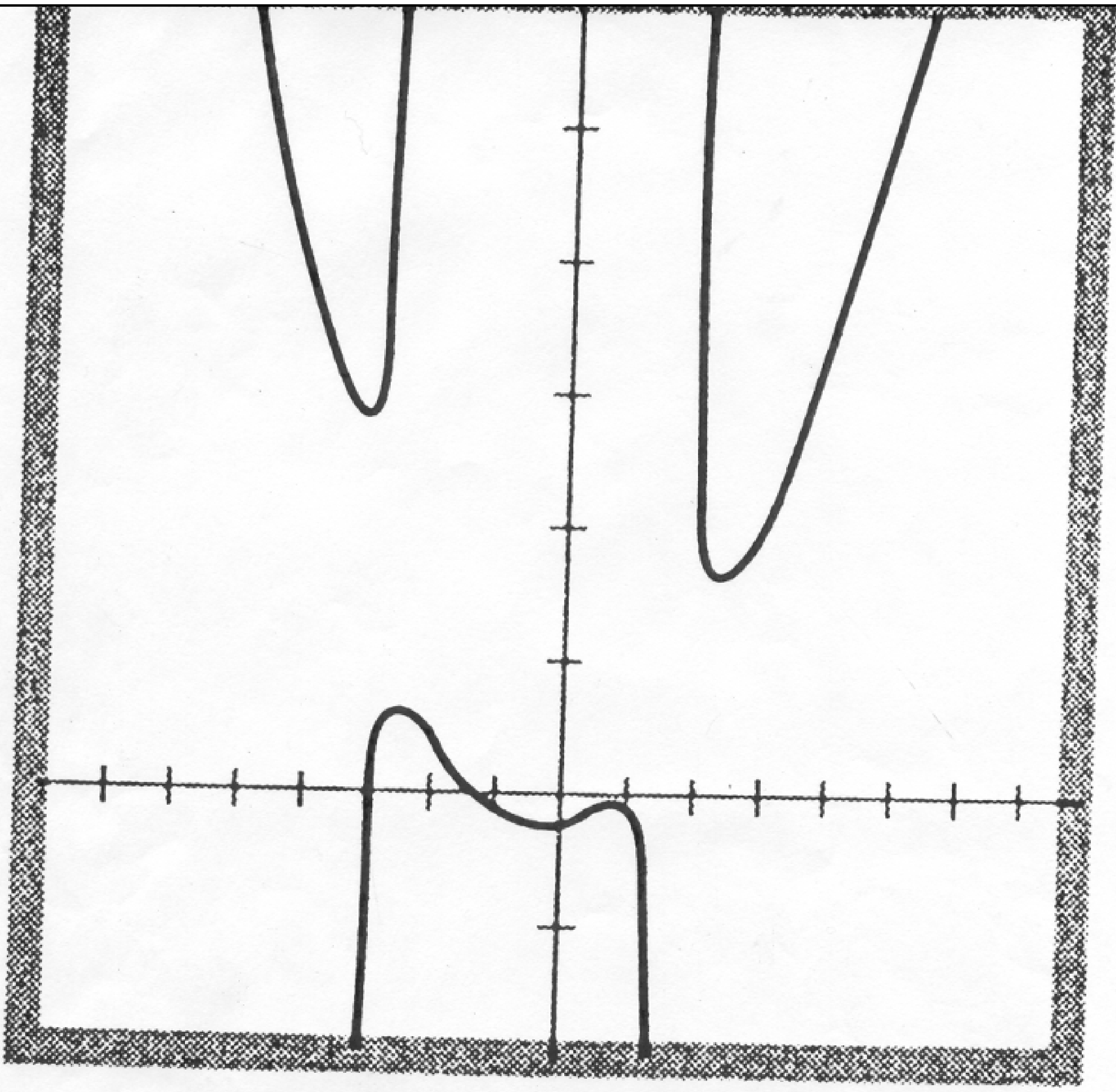
$$f(x) = \frac{p(x)}{h(x)} = q(x) + \frac{r(x)}{h(x)}$$

Where *q* is the quotient, *r* is the remainder and the *degree of r* < *degree of h*

The zeros of *h* give the Vertical Asymptotes.

and *q* is the End Behavior Asymptote

***this means that except near the Vertical Asymptotes, *q* is a very good approximation of *f*.



[−8, 8] by [−10, 30]

$$f(x) = \frac{x^4 + x^3 - 6x^2 + 6}{x^2 + x - 6}$$

Zeros at x = -2.85, x = -1

Points of discontinuity at x = 2, x = -3
These are Vertical Asymptotes
Function is differentiable everywhere else.

x^2 is the End Behavior Asymptote.

Rising on 4 intervals

Falling on 4 intervals

3 local maximums

2 local minimums

Concave Down over 3 intervals

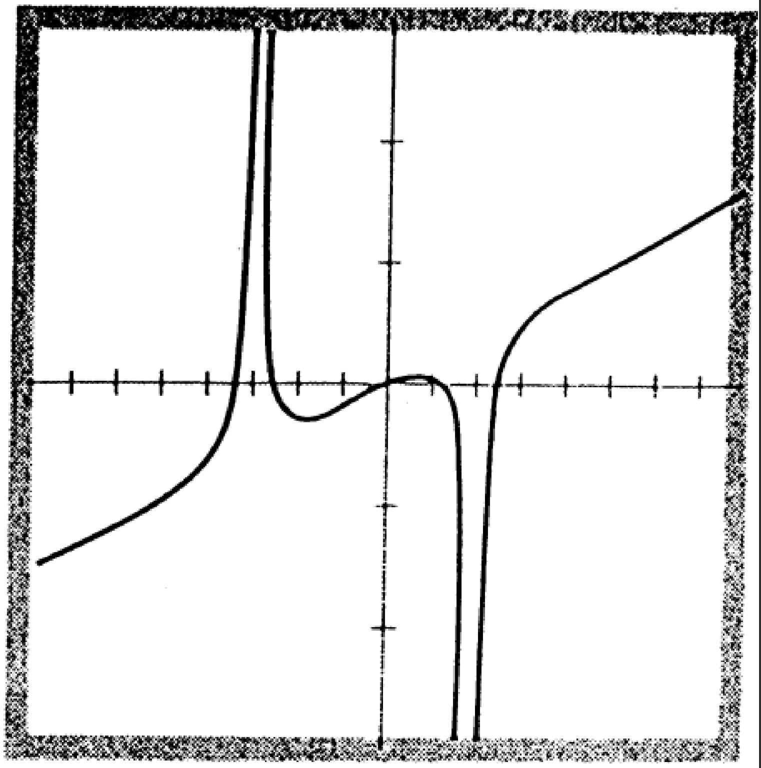
Concave Up over 2 intervals

2 Points of Inflection (both in the (-3, 2) interval).

$$f(x) = \frac{x^4 + x^3 - 6x^2 + 6}{x^2 + x - 6}$$

GRAPH of

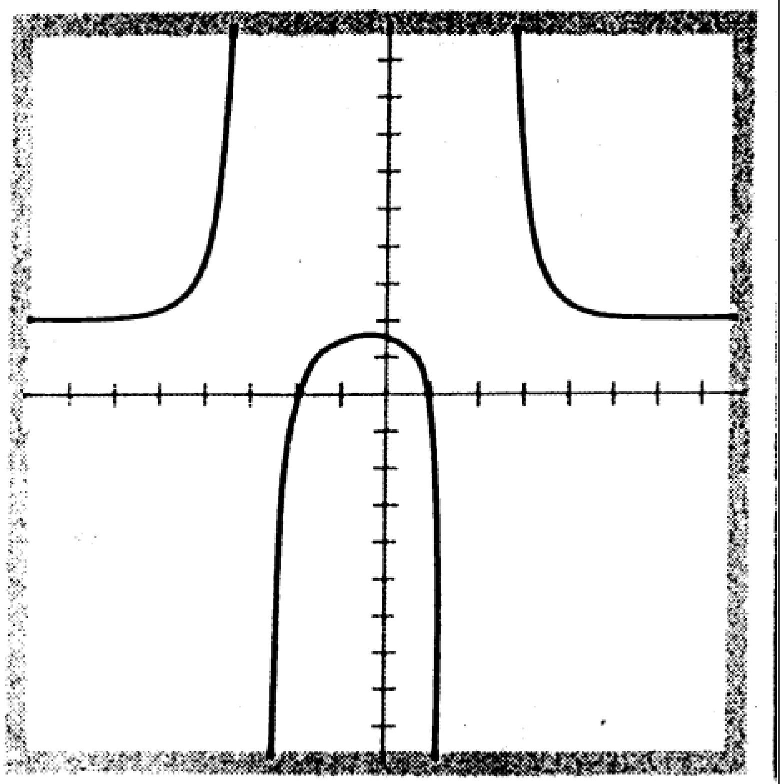
$f'(x)$ →



[−8, 8] by [−30, 30]

GRAPH OF

$f''(x)$ →



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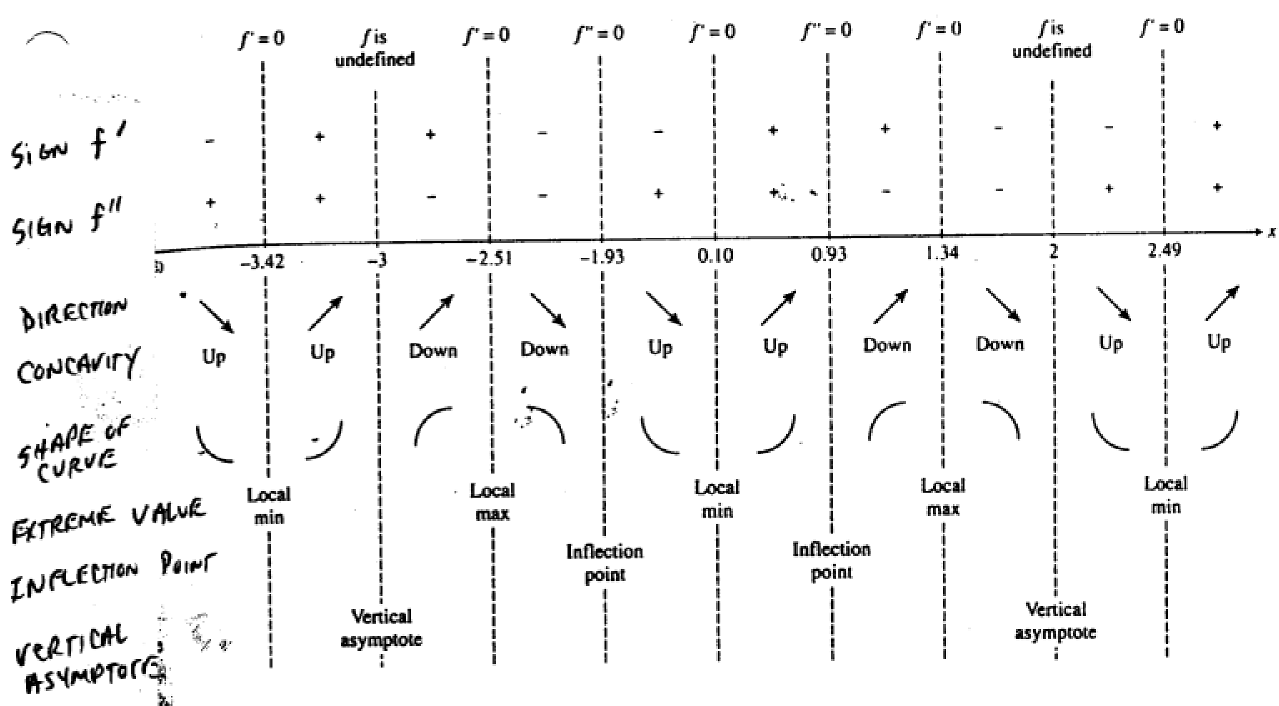


We can confirm the information in the chart by calculating

$$f'(x) = 2x - \frac{6(2x+1)}{(x^2+x-6)^2}$$

and

$$f''(x) = 2 + \frac{12(3x^2+3x+7)}{(x^2+x-6)^3}$$

A Complete Graph of a Rational Function f

To determine a complete graph of a rational function f, we do the following:

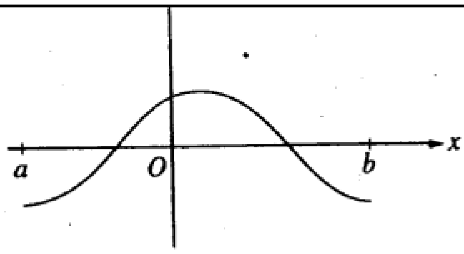
1. Establish the intercepts and domain of f, noting the vertical asymptotes. Find the limits from the left and right at each vertical asymptote. Find and visualize (or sketch) the end behavior asymptote.

2. Find f' and f''.

3. Find where f' is positive, negative, and zero to confirm where f is increasing and decreasing and has extreme values.

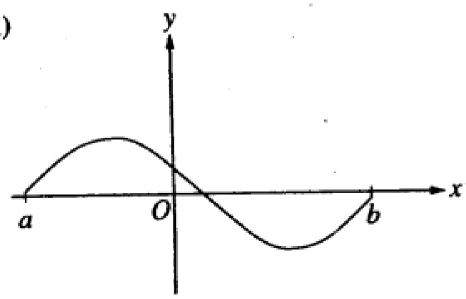
4. Find where f'' is positive, negative, and zero to determine where f is concave up, concave down and has points of inflection.

5. Compare the information found above with our graph and resolve conflicting information.

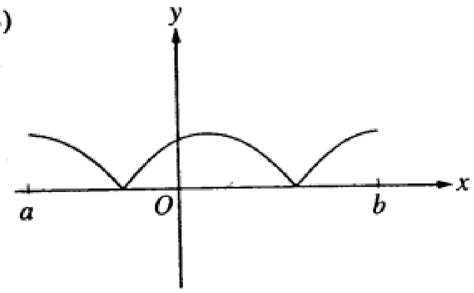


23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

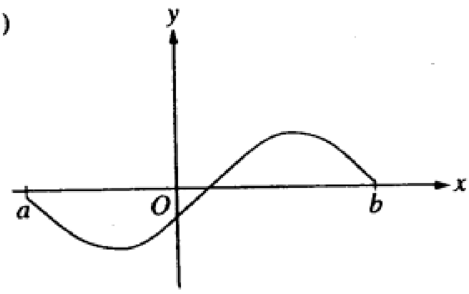
(A)



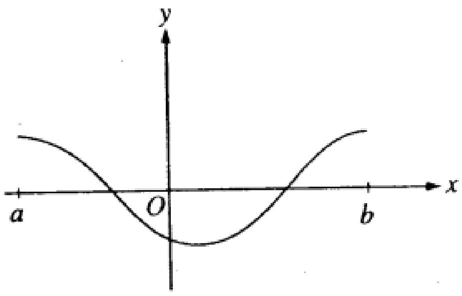
(B)



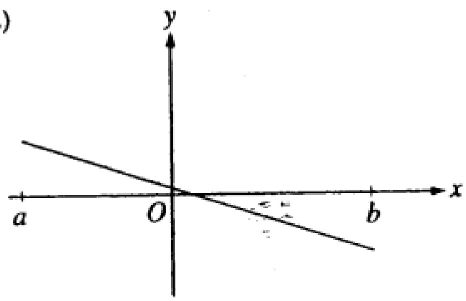
(C)



(D)



(E)



Design a one-liter oil can shaped like a right circular cylinder. What dimensions will use the least material? Justify your answer.

$V = \pi r^2 h = 1000$

$A = 2\pi r^2 + 2\pi rh$

Suppose a manufacturer can sell x items a week for a revenue of $r(x) = 200x - .01x^2$ cents

and it costs $c(x) = 50x + 20,000$ cents to make x items.

Is there a most profitable number of items to make each week? Find it and Justify your answer.

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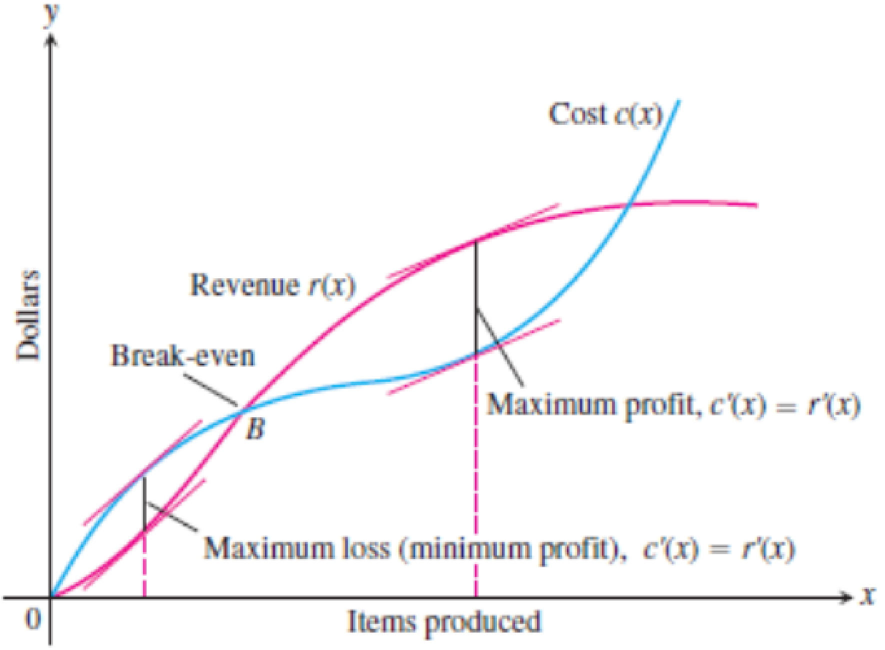
y
Dollars
Cost $c(x)$
Revenue $r(x)$
Break-even
B
Maximum profit, $c'(x) = r'(x)$
Maximum loss (minimum profit), $c'(x) = r'(x)$
0
Items produced
x

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Theorem 7

Maximum Profit (if any) occurs at a production level at which marginal revenue equals marginal cost.

Maximum Profit

$$R'(x) = C'(x)$$

Suppose $r(x) = 10x$ and $c(x) = x^3 - 6x^2 + 15x + 5$
where x represents thousands of units.

Is there a production level that maximizes profit?
Find it and justify your answer.

You now have all of the tools necessary to solve:

P. 305: 3, 5, 21, 27, 37, 42

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