CHAPTER 3 POLYNOMIAL AND RATIONAL FUNCTIONS

Section 3.1 Quadratic Functions and Models

Match each equation with the description of the parabola that is its graph.

1. $y=(x+3)^2-2$
2. $y=-(x-2)^2+3$
3. $y=-2(x-3)^2-2$
4. $y=2(x+3)^2+2$

a. Vertex (–3, –2), opens up

b. Vertex (3, –2), opens down

c. Vertex (–3, 2), opens up

d. Vertex (2, 3), opens down

Graph each parabola. Give the vertex, axis, domain, and range.

5. $y=-\frac{1}{2}x^2$

6. $y=2x^2$

7. $y=x^2-5$

8. $y=-(x-3)^2$

9. $y=\frac{1}{3}(x+3)^2$

10. $y=(x-1)^2-1$

11. $y=\frac{1}{2}(x+1)^2-2$

12. $y=2(x-1)^2+3$

13. $y=x^2-6x+5$

14. $y=-2x^2-16x-34$

15. $y=-x^2+8x-10$

16. $y=x^2-3x+2$

Solve each problem.

17. Find the dimensions of a rectangular field of maximum area that can be enclosed with 240 m of fencing if no fencing is needed on one side of the field.

18. If an object is thrown upward with an initial velocity of 64 ft per sec, then its height after t seconds is given by $h=64t-16t^2$. Find the maximum height above the ground and the number of seconds it takes the object to reach the maximum height.

19. A wire 48 cm long is cut into two pieces and each piece is bent to form a square. How long should each piece be to minimize the sum of the areas of the two squares?

20. A baseball is hit vertically at a velocity of 50 ft per sec. What is the maximum height that the ball will reach, and after how many seconds does the ball reach its maximum height? (The height after t seconds is given by $h=50t-16t^2$.)

Section 3.2 Synthetic Division

Use synthetic division to perform each division.

21. $\dfrac{2m^2+9m-35}{m+7}$

22. $\dfrac{2t^2-11t-21}{t-1}$

23. $\dfrac{k^3-8k^2+6k-3}{k-3}$

24. $\dfrac{2n^3+15n^2+28n}{n+4}$

25. $\dfrac{2r^3+13r^2+30r+20}{r+3}$

26. $\dfrac{3x^3+11x^2+11x+15}{x+3}$

27. $\dfrac{2x^4-2x^3+x-5}{x-3}$

28. $\dfrac{2x^4-5x^2+1}{x+2}$

29. $\dfrac{x^5-1024}{x-4}$

30. $\dfrac{x^5+243}{x+3}$

31. $\dfrac{3x^4-7x^3-x^2-\frac{1}{3}x-\frac{2}{9}}{x-\frac{1}{3}}$

32. $\dfrac{-6x^3+\frac{1}{2}x^2-\frac{1}{8}}{x+\frac{1}{2}}$

Express each polynomial in the form $f(x)=(x-k)q(x)+r$ for the given value of k.

33. $f(x)=12x^2-20x+3;\ k=3$

34. $f(x)=x^3-1;\ k=-1$

35. $f(x)=3x^3-4x^2+x-5;\ k=2$

36. $f(x)=x^4-256;\ k=-4$

37. $f(x)=5x^3-11x^2-10x;\ k=3$

38. $f(x)=2x^3-7x^2-17x-10;\ k=5$

39 $f(x)=x^4+x^2-1;\ k=-2$

40. $f(x)=2x^5-3x^4+x^2-4;\ k=-3$

For each polynomial, use the remainder theorem and synthetic division to find $f(k)$.

41. $k=4;\ f(x)=2x^2-3x+5$

42. $k=-3;\ f(y)=y^4-5y^2+1$

43. $k=-1;\ f(x)=x^3+3x^2-2x+1$

44. $k=2;\ f(s)=3s^3-2s^2-s+8$

45. $k=-2;\ f(r)=-r^3+2r^2-3r-22$

46. $k=-1;\ f(x)=x^4-3x^3+x^2+x-6$

47. $k=1+i;\ f(x)=x^2+4x-5$

48. $k=-2+3i;\ f(x)=x^3+2x^2-1$

Use synthetic division to decide whether the given number is a zero of the given polynomial.

49. 4; $f(y)=3y^2-10y-8$

50. -3; $f(z)=2z^3+13z^2+30z+25$

51. 3; $f(m)=10m^4-37m^3+34m^2-16m+15$

52. 2; $f(r)=3r^4-6r^2-r-14$

53. $1+i$; $f(x)=x^2-2x+2$

54. i; $f(x)=2x^3+ix^2-3x+1$

55. $2-i$; $f(r)=r^3-2r^2+r-1$

56. $3+2i$; $f(x)=x^3-8x^2+25x-26$

Use the factor theorem to decide whether the second polynomial is a factor of the first.

57. $2x^3 - x^2 + 3x + 4;\ x + 1$

58. $x^4 - 10x^3 + 35x^2 - 50x + 24;\ x - 4$

59. $x^4 + 12x^3 + 35x^2 + 24x;\ x - 8$

60. $x^3 + 4x^2 + x - 6;\ x - 1$

61. $x^4 - 5x^2 + 4;\ x + 1$

62. $4x^3 - 8x^2 - x + 2;\ x - \frac{1}{2}$

For each polynomial, one zero is given. Find all others.

63. $f(x) = x^4 - x^3 - 7x^2 + x + 6;\ 1$

64. $f(x) = x^4 + 6x^3 + 7x^2 - 6x - 8;\ 1$

65. $f(x) = x^2 - 4x + 8;\ 2 + 2i$

66. $f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12;\ 2$

67. $f(x) = x^4 - 4x^3 + 7x^2 - 16x + 12;\ 2i$

68. $f(x) = x^4 + 6x^3 + 18x^2 + 14x - 39;\ -2 - 3i$

For the given information, find a polynomial of degree 3 with only real coefficients that satisfies the given conditions.

69. Zeros of 1, –2, and 3; $f(2) = 8$

70. Zeros of –3, 2, and 8; $f(1) = 28$

71. Zeros of 3, i, and $-i$; $f(2) = 20$

72. Zeros of –6, i, and $-i$; $f(-3) = 60$

Factor $f(x)$ into linear factors given that k is a zero of $f(x)$.

73. $f(x) = x^3 - 7x^2 - 25x + 175;\ k = 5$

74. $f(x) = x^3 - 4x^2 + 9x - 10;\ k = 2$

75. $f(x) = x^4 - 6x^3 + 11x^2 - 6x + 10;\ k = -i$

76. $f(x) = x^3 + (6 - 2i)x^2 + (8 - 12i)x - 16i;\ k = 2i$

For each polynomial function, find all zeros and their multiplicities.

77. $f(x) = x^4(x-2)^2(x+1)$

78. $f(x) = (x+5)^4(x-4)^5$

79. $f(x) = (x+i)^3(x-i)^3$

80. $f(x) = (x-3)(x-4i)^5(x+4i)^5$

For each of the following, find a polynomial of lowest degree with only real coefficients and having the given zeros.

81. 9, 1, 2, and –2

82. $-\sqrt{5}, \sqrt{5}$, and 8

83. $1-\sqrt{7}, 1+\sqrt{7}, \sqrt{3}$, and $-\sqrt{3}$

84. $2i, -2i, 5$

85. $2, i\sqrt{3}$

86. $-i, 3 + i$

Section 3.4 Polynomial Functions: Graphs, Applications, and Models

Sketch the graph of each polynomial function.

87. $f(x) = 2x^3$

88. $f(x) = -2x^4$

89. $f(x) = 3x^5$

90. $f(x) = \frac{1}{3}x^3$

91. $f(x) = \frac{1}{2}x^4$

92. $f(x) = -\frac{1}{3}x^5$

93. $f(x) = -\frac{3}{4}x^3 - 1$

94. $f(x) = \frac{1}{2}(x-1)^4$

95. $f(x) = 3(x-2)^3$

96. $f(x) = (x+1)^4 + 2$

Graph each polynomial function. Factor first if the expression is not in factored form.

~~97. $f(x) = x(2x-1)(x+1)$~~

~~98. $f(x) = x^2(3x-1)(4x+3)$~~

99. $f(x) = 3x^3 + 8x^2 + 4x$

100. $f(x) = x(x+1)(x-2)(x+3)$

101. $f(x) = x^3 - 6x^2 + 8x$

102. $f(x) = -x^4 + 4x^2$

103. $f(x) = 4x^4 + 3x^3 - 10x^2$

104. $f(x) = x^4 - 5x^2 + 4$

105. $f(x) = 2x^3 + x^2 - 8x - 4$

106. $f(x) = -x^3 + 2x^2 + x - 2$

Use the intermediate value theorem for polynomials to show that each function has a real zero between the numbers given.

107. $f(x) = 3x^3 - 6x^2 + x + 1$; -1 and 0

108. $f(x) = x^3 + 3x^2 - 18x - 10$; -6 and -5

109. $f(x) = 5x^3 - x^2 - 25x + 5$; -3 and -2

110. $f(x) = 6x^3 - 37x^2 + 48x - 7$; 4 and 5

Show that the real zeros of each polynomial satisfy the given conditions.

111. $f(x) = 3x^3 - 3x^2 - 18x - 2$; no real zero greater than 4

112. $f(x) = 5x^4 + 3x^2 + 2x - 3$; no real zero greater than 1

~~113. $f(x) = 2x^3 + 2x^2 - 4x + 2$; no real zero less than -3~~

114. $f(x) = x^5 - 3x^3 - x + 2$; no real zero less than -2

For the given polynomial, approximate each real zero as a decimal to the nearest tenth.

115. $f(x) = 2x^3 - 9x^2 - 3x + 4$

116. $f(x) = x^4 - 3x^3 - 4x^2 + 13x - 12$

117. $f(x) = x^3 - 4x^2 - 3x + 12$

118. $f(x) = x^3 - 10x^2 + 13x + 44$

119. $f(x) = 6x^4 - 7x^3 - 23x^2 + 14x + 3$

120. $f(x) = x^4 - 5x^2 + 6$

Use a graphing calculator to find the coordinates of the turning points of the graph of each polynomial function in the given interval. Give answers to the nearest hundredth.

121. $f(x) = 2x^3 - 9x^2 - 3x + 4$, $[3, 3.5]$

122. $f(x) = x^4 - 3x^3 - 4x^2 + 13x - 12$, $[-1.8, -1]$

123. $f(x) = x^3 - 6x^2 + 4x + 16$, $[3, 4]$

124. $f(x) = x^4 - 4x^3 - 4x^2 + 16x + 12$, $[-1.8, -1]$

125. $f(x) = x^3 - 15x^2 + 73x - 115$, $[4, 4.8]$

126. $f(x) = x^3 - 6x^2 + 16$, $[3.5, 4.3]$

Use reflections, symmetry, and translations to graph each rational function.

127. $f(x) = -\dfrac{4}{x}$

128. $f(x) = \dfrac{1}{x+3}$

129. $f(x) = \dfrac{1}{x} + 2$

130. $f(x) = 2 - \dfrac{1}{x}$

Give the equations of the vertical, horizontal, and/or oblique asymptotes of each rational function.

131. $f(x) = -\dfrac{5}{x}$

132. $f(x) = \dfrac{3}{4x-1}$

133. $f(x) = \dfrac{2x+1}{x-3}$

134. $f(x) = \dfrac{4x+2}{x^2+6x+5}$

135. $f(x) = \dfrac{4}{(x-2)^2}$

136. $f(x) = \dfrac{x}{x^2-4}$

137. $f(x) = \dfrac{(x-4)(x+3)}{(x+1)(x-5)}$

138. $f(x) = \dfrac{x^2-4}{x+1}$

139. $f(x) = \dfrac{x^2-3x+4}{x^2-2x+1}$

140. $f(x) = \dfrac{x^2+3x+2}{x-1}$

Sketch the graph of each rational function.

141. $f(x) = \dfrac{3}{2x+1}$

142. $f(x) = \dfrac{4x}{(x-2)(x+1)}$

143. $f(x) = \dfrac{1}{(x+2)^2}$

144. $f(x) = \dfrac{3x}{(x-1)(x+2)}$

145. $f(x) = \dfrac{x+1}{x^2-9}$

146. $f(x) = \dfrac{2x-1}{4x+3}$

147. $f(x) = \dfrac{2-x}{x}$

148. $f(x) = \dfrac{x^2(x^2-4)}{(x^2-1)^2}$

149. $f(x) = \dfrac{x^2-4}{x^2-1}$

150. $f(x) = \dfrac{(x-3)(x+2)}{(x+4)(x-1)}$

151. $f(x) = \dfrac{x^2-4}{x}$

152. $f(x) = \dfrac{x^2+2}{x+1}$

153. $f(x) = \dfrac{-x^2+3x+2}{x-4}$

154. $f(x) = \dfrac{x^3+1}{x^2}$

155. $f(x) = \dfrac{x^2-9}{x+3}$

156. $f(x) = \dfrac{x^2+4x+4}{x+2}$

157. $f(x) = \dfrac{x^3}{x}$

158. $f(x) = \dfrac{x^3+7x^2+14x+8}{x+1}$

Section 3.6 Variation

Solve each variation problem.

161. If r varies directly as t, and $r = 10$ when $t = 2$, find r when $t = 9$.

162. If q varies directly as p, and $q = 36$ when $p = 5$, find q when $p = 20$.

163. If m varies directly as p^2, and $m = 20$ when $p = 2$, find m when $p = 5$.

164. If a varies directly as b^2, and $a = 48$ when $b = 4$, find a when $b = 7$.

165. If y varies directly as the square of z and $y = 8$ when $z = 6$, find y when $z = 9$.

166. If y varies inversely as x, and $y = 10$ when $x = 3$, find y when $x = 12$.

167. If r varies inversely as s and $r = 7$ when $s = 8$, find r when $s = 12$.

168. If r varies inversely as t^2, and $r = 8$ when $t = 4$, find r when $t = 9$.

169. If p varies inversely as q^2, and $p = 4$, when $q = 1/2$, find p when $q = 3/2$.

170. If r varies jointly as m and n^2 and $r = 72$ when $m = 4$ and $n = 6$, find r when $m = 3$ and $n = 4$.

171. If q varies jointly as p and r^2, and $q = 27$ when $p = 9$ and $r = 2$, find q when $p = 8$ and $r = 4$.

172. If y varies jointly as x^2 and z^2, and $y = 72$ when $x = 2$ and $z = 3$, find y when $x = 4$ and $z = 2$.

173. The circumference of a circle varies directly as the radius. A circle with a radius of 7 cm has a circumference of 43.96 cm. Find the circumference of the circle if the radius changes to 11 cm.

174. For a body falling freely from rest (disregarding air resistance), the distance the body falls varies directly as the square of the time. If an object is dropped from a tower 400 ft high and hits the ground in 5 sec, how far did it fall in the first 3 sec?

175. The current in a simple electrical circuit varies inversely as the resistance. If the current is 50 amperes (an ampere is a unit for measuring current) when the resistance is 10 ohms (an ohm is a unit for measuring resistance), find the current if the resistance is 5 ohms.

176. The force required to compress a spring varies directly as the change in the length of spring. If a force of 12 lb is required to compress a certain spring 3 in., how much force is required to compress the spring 5 in.?

177. The illumination produced by a light source varies inversely as the square of the distance from the source. If the illumination produced 4 ft from a light source is 75 foot-candles, find the illumination produced 9 ft from the same source.

178. The time required to print a newsletter varies directly as the number of newsletters printed and inversely as the number of presses used. If 40,000 newsletters are printed in 1 hr when two presses are used, how long will it take to print 30,000 newsletters if three presses are used?

179. When an object is moving in a circular path, the centripetal force varies directly as the square of the velocity and inversely as the radius of the circle. A stone that is whirled at the end of a string 50 cm long at 900 cm per sec has a centripetal force of 3,240,000 dynes. Find the centripetal force if the stone is whirled at the end of a string 75 cm long at 1500 cm per sec.

180. The gravitational attraction between two objects varies directly as the product of their masses and inversely as the square of the distance between them. If the force of attraction between two spheres, each with a mass of 1 g, that are 1 cm apart, is 6.66×10^{-8} dynes, find the force of attraction between two spheres with masses of 2 g and 4 g that are 4 cm apart.

181. The electrical resistance R of a wire varies directly as its length L and inversely as its cross-sectional area A. If the resistance of a wire is .2 ohm when the length is 200 ft and its cross-sectional area is .05 sq in., find the resistance of a wire whose length is 5000 ft with a cross-sectional area of .01 sq in.